Circular Motion and Gravitation

Problem A

CENTRIPETAL ACCELERATION

**Problem**
Calculate the orbital radius of the Earth, if its tangential speed is 29.7 km/s and the centripetal acceleration acting on Earth is $5.9 \times 10^{-3}$ m/s².

**Solution**

Given:  
$v_t = 29.7$ km/s  
$a_c = 5.9 \times 10^{-3}$ m/s²

Unknown:  
$r = ?$

Use the centripetal acceleration equation written in terms of tangential speed. Rearrange the equation to solve for $r$.

$$a_c = \frac{v_t^2}{r}$$

$$r = \frac{v_t^2}{a_c} = \frac{(29.7 \times 10^3 \text{ m/s})^2}{5.9 \times 10^{-3} \text{ m/s}^2} = 1.5 \times 10^{11} \text{ m} = 1.5 \times 10^8 \text{ km}$$

**Additional Practice**

1. The largest salami in the world, made in Norway, was more than 20 m long. If a hungry mouse ran around the salami’s circumference with a tangential speed of 0.17 m/s, the centripetal acceleration of the mouse was 0.29 m/s². What was the radius of the salami?

2. An astronomer at the equator measures the Doppler shift of sunlight at sunset. From this, she calculates that Earth’s tangential velocity at the equator is 465 m/s. The centripetal acceleration at the equator is $3.41 \times 10^{-2}$ m/s². Use this data to calculate Earth’s radius.

3. In 1994, Susan Williams of California blew a bubble-gum bubble with a diameter of 58.4 cm. If this bubble were rigid and the centripetal acceleration of the equatorial points of the bubble were $8.50 \times 10^{-2}$ m/s², what would the tangential speed of those points be?

4. An ostrich lays the largest bird egg. A typical diameter for an ostrich egg at its widest part is 12 cm. Suppose an egg of this size rolls down a slope so that the centripetal acceleration of the shell at its widest part is 0.28 m/s². What is the tangential speed of that part of the shell?

5. A waterwheel built in Hamah, Syria, has a radius of 20.0 m. If the tangential velocity at the wheel’s edge is 7.85 m/s, what is the centripetal acceleration of the wheel?

6. In 1995, Cathy Marsal of France cycled 47.112 km in 1.000 hour. Calculate the magnitude of the centripetal acceleration of Marsal with respect to Earth’s center. Neglect Earth’s rotation, and use $6.37 \times 10^3$ km as Earth’s radius.
Circular Motion and Gravitation

Problem B

CENTRIPETAL FORCE

PROBLEM
The royal antelope of western Africa has an average mass of only 3.2 kg. Suppose this antelope runs in a circle with a radius of 30.0 m. If a force of 8.8 N maintains this circular motion, what is the antelope’s tangential speed?

SOLUTION
Given:
- \( m = 3.2 \) kg
- \( r = 30.0 \) m
- \( F_c = 8.8 \) N

Unknown: \( v_t \)

Use the equation for centripetal force, and rearrange it to solve for tangential speed.

\[
F_c = \frac{mv_t^2}{r}
\]

\[
v_t = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{(8.8 \text{ N})(30.0 \text{ m})}{3.2 \text{ kg}}} = \sqrt{\frac{82 \text{ m}^2}{s^2}}
\]

\[v_t = 9.1 \text{ m/s}\]

ADDITIONAL PRACTICE

1. Gregg Reid of Atlanta, Georgia, built a motorcycle that is over 4.5 m long and has a mass of 235 kg. The force that holds Reid and his motorcycle in a circular path with a radius 25.0 m is 1850 N. What is Reid’s tangential speed? Assume Reid’s mass is 72 kg.

2. With an average mass of only 30.0 g, the mouse lemur of Madagascar is the smallest primate on Earth. Suppose this lemur swings on a light vine with a length of 2.4 m, so that the tension in the vine at the bottom point of the swing is 0.393 N. What is the lemur’s tangential speed at that point?

3. In 1994, Mata Jagdamba of India had very long hair. It was 4.23 m long. Suppose Mata conducted experiments with her hair. First, she determined that one hair strand could support a mass of 25 g. She then attached a smaller mass to the same hair strand and swung it in the horizontal plane. If the strand broke when the tangential speed of the mass reached 8.1 m/s, how large was the mass?

4. Pat Kinch used a racing cycle to travel 75.57 km/h. Suppose Kinch moved at this speed around a circular track. If the combined mass of Kinch and the cycle was 92.0 kg and the average centripetal force was 12.8 N, what was the radius of the track?
5. In 1992, a team of 12 athletes from Great Britain and Canada rappelled 446 m down the CN Tower in Toronto, Canada. Suppose an athlete with a mass of 75.0 kg, having reached the ground, took a joyful swing on the 446 m-long rope. If the speed of the athlete at the bottom point of the swing was 12 m/s, what was the centripetal force? What was the tension in the rope? Neglect the rope's mass.
Circular Motion and Gravitation

Problem C

GRAVITATIONAL FORCE

PROBLEM

The sun has a mass of $2.0 \times 10^{30}$ kg and a radius of $7.0 \times 10^{5}$ km. What mass must be located at the sun’s surface for a gravitational force of $470$ N to exist between the mass and the sun?

SOLUTION

Given: 
- $m_1 = 2.0 \times 10^{30}$ kg
- $r = 7.0 \times 10^{5}$ km = $7.0 \times 10^{8}$ m
- $G = 6.673 \times 10^{-11}$ N\(\cdot\)m$^2$/kg$^2$
- $F_g = 470$ N

Unknown: 
- $m_2 = ?$

Use Newton’s universal law of gravitation, and rearrange it to solve for the second mass.

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$m_2 = \frac{F_g r^2}{G m_1} = \frac{(470\text{ N})(7.0 \times 10^{8}\text{ m})^2}{6.673 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2}(2.0 \times 10^{30}\text{ kg})$$

$$m_2 = 1.7\text{ kg}$$

ADDITIONAL PRACTICE

1. Deimos, a satellite of Mars, has an average radius of 6.3 km. If the gravitational force between Deimos and a 3.0 kg rock at its surface is $2.5 \times 10^{-2}$ N, what is the mass of Deimos?

2. A $3.08 \times 10^4$ kg meteorite is on exhibit in New York City. Suppose this meteorite and another meteorite are separated by $1.27 \times 10^7$ m (a distance equal to Earth’s average diameter). If the gravitational force between them is $2.88 \times 10^{-16}$ N, what is the mass of the second meteorite?

3. In 1989, a cake with a mass of $5.81 \times 10^4$ kg was baked in Alabama. Suppose a cook stood 25.0 m from the cake. The gravitational force exerted between the cook and the cake was $5.0 \times 10^{-7}$ N. What was the cook’s mass?

4. The largest diamond ever found has a mass of 621 g. If the force of gravitational attraction between this diamond and a person with a mass of 65.0 kg is $1.0 \times 10^{-12}$ N, what is the distance between them?
5. The passenger liners *Carnival Destiny* and *Grand Princess*, built recently, have a mass of about $1.0 \times 10^8$ kg each. How far apart must these two ships be to exert a gravitational attraction of $1.0 \times 10^{-3}$ N on each other?

6. In 1874, a swarm of locusts descended on Nebraska. The swarm’s mass was estimated to be $25 \times 10^9$ kg. If this swarm were split in half and the halves separated by $1.0 \times 10^3$ km, what would the magnitude of the gravitational force between the halves be?

7. Jupiter, the largest planet in the solar system, has a mass 318 times that of Earth and a volume that is 1323 times greater than Earth’s. Calculate the magnitude of the gravitational force exerted on a 50.0 kg mass on Jupiter’s surface.
Advanced Topics

**Problem A**

**ANGULAR DISPLACEMENT**

**PROBLEM**

The diameter of the outermost planet, Pluto, is just \(2.30 \times 10^3\) km. If a space probe were to orbit Pluto near the planet’s surface, what would the arc length of the probe’s displacement be after it had completed eight orbits?

**SOLUTION**

Given:

\[
\begin{align*}
  r &= \frac{2.30 \times 10^3\, \text{km}}{2} = 1.15 \times 10^3\, \text{km} \\
  \Delta \theta &= 8 \text{ orbits} = 8(2\pi\, \text{rad}) = 16\pi\, \text{rad}
\end{align*}
\]

Unknown:

\(\Delta s = ?\)

Use the angular displacement equation and rearrange to solve for \(\Delta s\).

\[
\Delta s = r \Delta \theta = (1.15 \times 10^3\, \text{km})(16\pi\, \text{rad}) = 5.78 \times 10^4\, \text{km}
\]

**ADDITIONAL PRACTICE**

1. A neutron star can have a mass three times that of the sun and a radius as small as 10.0 km. If a particle travels +15.0 rad along the equator of a neutron star, through what arc length does the particle travel? Does the particle travel in the clockwise or counterclockwise direction from the viewpoint of the neutron star’s “north” pole?

2. John Glenn was the first American to orbit Earth. In 1962, he circled Earth counterclockwise three times in less that 5 h aboard his spaceship *Friendship 7*. If his distance from the Earth’s center was 6560 km, what arc length did Glenn and his spaceship travel through?

3. Jupiter’s diameter is \(1.40 \times 10^5\) km. Suppose a space vehicle travels along Jupiter’s equator with an angular displacement of 1.72 rad.
   a. Through what arc length does the space vehicle move?
   b. How many orbits around Earth’s equator can be completed if the vehicle travels at the surface of the Earth through this arc length around Earth? Use \(6.37 \times 10^3\) km for Earth’s radius.

4. In 1981, the space shuttle *Columbia* became the first reusable spacecraft to orbit Earth. The shuttle’s total angular displacement as it orbited Earth was 225 rad. How far from Earth’s center was *Columbia* if it moved through an arc length of \(1.50 \times 10^6\) km? Use \(6.37 \times 10^3\) km for Earth’s radius.

5. Mercury, the planet closest to the sun, has an orbital radius of \(5.8 \times 10^7\) km. Find the angular displacement of Mercury as it travels through an arc length equal to the radius of Earth’s orbit around the sun \((1.5 \times 10^8\, \text{km})\).
6. From 1987 through 1988, Sarah Covington-Fulcher ran around the United States, covering a distance of $1.79 \times 10^4$ km. If she had run that distance clockwise around Earth's equator, what would her angular displacement have been? (Earth's average radius is $6.37 \times 10^3$ km).
Problem B

ANGULAR VELOCITY

PROBLEM

In 1975, an ultrafast centrifuge attained an average angular speed of $2.65 \times 10^4$ rad/s. What was the centrifuge’s angular displacement after 1.5 s?

SOLUTION

Given:

- $\omega_{\text{avg}} = 2.65 \times 10^4$ rad/s
- $\Delta t = 1.5$ s

Unknown:

- $\Delta \theta = ?$

Use the angular speed equation and rearrange to solve for $\Delta \theta$.

\[
\omega_{\text{avg}} = \frac{\Delta \theta}{\Delta t}
\]

\[
\Delta \theta = \omega_{\text{avg}} \Delta t = (2.65 \times 10^4 \text{ rad/s})(1.5 \text{ s})
\]

\[
\Delta \theta = 4.0 \times 10^4 \text{ rad}
\]

ADDITIONAL PRACTICE

1. The largest steam engine ever built was constructed in 1849. The engine had one huge cylinder with a radius of 1.82 m. If a beetle were to run around the edge of the cylinder with an average angular speed of $1.00 \times 10^{-1}$ rad/s, what would its angular displacement be after 60.0 s? What arc length would it have moved through?

2. The world’s largest planetarium dome is 30 m in diameter. What would your angular displacement be if you ran around the perimeter of this dome for 120 s with an average angular speed of 0.40 rad/s?

3. The world’s most massive magnet is located in a research center in Dubna, Russia. This magnet has a mass of over $3.0 \times 10^7$ kg and a radius of 30.0 m. If you were to run around this magnet so that you traveled $5.0 \times 10^2$ m in 120 s, what would your average angular speed be?

4. A floral clock in Japan has a radius of 15.5 m. If you ride a bike around the clock, making 16 revolutions in 4.5 min, what is your average angular speed?

5. A revolving globe with a radius of 5.0 m was built between 1982 and 1987 in Italy. If the globe revolves with the same average angular speed as Earth, how long will it take for a point on the globe’s equator to move through 0.262 rad?

6. A water-supply tunnel in New York City is $1.70 \times 10^2$ km long and has a radius of 2.00 m. If a beetle moves around the tunnel’s perimeter with an average angular speed of 5.90 rad/s, how long will it take the beetle to travel a distance equal to the tunnel’s length?
Advanced Topics

Problem C

ANGULAR ACCELERATION

PROBLEM

A self-propelled Catherine wheel (a spinning wheel with fireworks along its rim) with a diameter of 20.0 m was built in 1994. Its maximum angular speed was 0.52 rad/s. How long did the wheel undergo an angular acceleration of $2.6 \times 10^{-2}$ rad/s$^2$ in order to reach its maximum angular speed?

SOLUTION

Given:

| $\omega_1$ | 0 rad/s |
| $\omega_2$ | 0.52 rad/s |
| $\alpha_{avg}$ | $2.6 \times 10^{-2}$ rad/s$^2$ |

Unknown: $\Delta t = ?$

Use the angular acceleration equation and rearrange to solve for $\Delta t$.

$$\alpha_{avg} = \frac{\Delta \omega}{\Delta t}$$

$$\Delta t = \frac{\Delta \omega}{\alpha_{avg}} = \frac{\omega_2 - \omega_1}{\alpha_{avg}} = \frac{0.52 \text{ rad/s} - 0 \text{ rad/s}}{2.6 \times 10^{-2} \text{ rad/s}^2} = 2.0 \times 10^1 \text{ s}$$

ADDITIONAL PRACTICE

1. Peter Rosendahl of Sweden rode a unicycle with a wheel diameter of 2.5 cm. If the wheel’s average angular acceleration was 2.0 rad/s$^2$, how long would it take for the wheel’s angular speed to increase from 0 rad/s to 9.4 rad/s?

2. Jupiter has the shortest day of all of the solar system’s planets. One rotation of Jupiter occurs in 9.83 h. If an average angular acceleration of $-3.0 \times 10^{-8}$ rad/s$^2$ slows Jupiter’s rotation, how long does it take for Jupiter to stop rotating?

3. In 1989, Dave Moore of California built the Frankencycle, a bicycle with a wheel diameter of more than 3 m. If you ride this bike so that the wheels’ angular speed increases from 2.00 rad/s to 3.15 rad/s in 3.6 s, what is the average angular acceleration of the wheels?

4. In 1990, David Robilliard rode a bicycle on the back wheel for more than 5 h. If the wheel’s initial angular speed was 8.0 rad/s and Robilliard tripled this speed in 25 s, what was the average angular acceleration?

5. Earth takes about 365 days to orbit once around the sun. Mercury, the innermost planet, takes less than a fourth of this time to complete one revolution. Suppose some mysterious force causes Earth to experience an average angular acceleration of $6.05 \times 10^{-13}$ rad/s$^2$, so that after
12.0 days its angular orbital speed is the same as that of Mercury. Calculate this angular speed and the period of one orbit.

6. The smallest ridable tandem bicycle was built in France and has a length of less than 40 cm. Suppose this bicycle accelerates from rest so that the average angular acceleration of the wheels is 0.800 rad/s². What is the angular speed of the wheels after 8.40 s?
Advanced Topics

Problem D

ANGULAR KINEMATICS

PROBLEM

In 1990, a pizza with a radius of 18.7 m was baked in South Africa. Suppose this pizza was placed on a rotating platform. If the pizza accelerated from rest at 5.00 rad/s² for 25.0 s, what was the pizza's final angular speed?

SOLUTION

Given: 
- \( \omega_i = 0 \text{ rad/s} \)
- \( \alpha = 5.00 \text{ rad/s}^2 \)
- \( \Delta t = 25.0 \text{ s} \)

Unknown: 
- \( \omega_f = ? \)

Use the rotational kinematic equation relating final angular speed to initial angular speed, angular acceleration, and time.

\[ \omega_f = \omega_i + \alpha \Delta t \]

\[ \omega_f = 0 \text{ rad/s} + (5.00 \text{ rad/s}^2)(25.0 \text{ s}) \]

\[ \omega_f = 125 \text{ rad/s} \]

ADDITIONAL PRACTICE

1. In 1987, Takayuki Koike of Japan rode a unicycle nonstop for 160 km in less than 7 h. Suppose at some point in his trip Koike accelerated downhill. If the wheel's angular speed was initially 5.0 rad/s, what would the angular speed be after the wheel underwent an angular acceleration of 0.60 rad/s² for 0.50 min?

2. The moon orbits Earth in 27.3 days. Suppose a spacecraft leaves the moon and follows the same orbit as the moon. If the spacecraft has a constant angular acceleration of \( 1.0 \times 10^{-10} \text{ rad/s}^2 \), what is its angular speed after 12 h of flight? (Hint: At takeoff, the spaceship has the same angular speed as the moon.)

3. African baobab trees can have circumferences of up to 43 m. Imagine riding a bicycle around a tree this size. If, starting from rest, you travel a distance of 160 m around the tree with a constant angular acceleration of \( 5.0 \times 10^{-2} \text{ rad/s}^2 \), what will your final angular speed be?

4. In 1976, Kathy Wafler produced an unbroken apple peel 52.5 m in length. Suppose Wafler turned the apple with a constant angular acceleration of \( -3.2 \times 10^{-5} \text{ rad/s}^2 \), until her final angular speed was 0.080 rad/s. Assuming the apple was a sphere with a radius of 8.0 cm, calculate the apple's initial angular speed.
5. In 1987, a giant hanging basket of flowers with a mass of 4000 kg was constructed. The radius of the basket was 3.0 m. Suppose this basket was placed on the ground and an admiring spectator ran around it to see every detail again and again. At first the spectator’s angular speed was 0.820 rad/s, but he steadily decreased his speed to 0.360 rad/s by the time he had traveled 20.0 m around the basket. Find the spectator’s angular acceleration.

6. One of the largest scientific devices in the world is the particle accelerator at Fermilab, in Batavia, Illinois. The accelerator consists of a giant ring with a radius of 1.0 km. Suppose a maintenance engineer drives around the accelerator, starting at an angular speed of $5.0 \times 10^{-3}$ rad/s and accelerating at a constant rate until one trip is completed in 14.0 min. Find the engineer’s angular acceleration.

7. With a diameter of 207.0 m, the Superdome in New Orleans, Louisiana, is the largest “dome” in the world. Imagine a race held along the Superdome’s outside perimeter. An athlete whose initial angular speed is $7.20 \times 10^{-2}$ rad/s runs 12.6 rad in 4 min 22 s. What is the athlete’s constant angular acceleration?

8. In 1986, Fred Markham rode a bicycle that was pulled by an automobile 200 m in 6.83 s. Suppose the angular speed of the bicycle’s wheels increased steadily from 27.0 rad/s to 32.0 rad/s. Find the wheels’ angular acceleration.

9. Consider a common analog clock. At midnight, the hour and minute hands coincide. Then the minute hand begins to rotate away from the hour hand. Suppose you adjust the clock by pushing the hour hand clockwise with a constant acceleration of $2.68 \times 10^{-5}$ rad/s$^2$. What is the angular displacement of the hour hand after 120.0 s? (Note that the unaccelerated hour hand makes one full rotation in 12 h.)

10. Herman Bax of Canada grew a pumpkin with a circumference of 7 m. Suppose an ant crawled around the pumpkin’s “equator.” The ant started with an angular speed of $6.0 \times 10^{-3}$ rad/s and accelerated steadily at a rate of $2.5 \times 10^{-4}$ rad/s$^2$ until its angular speed was tripled. What was the ant’s angular displacement?

11. Tal Burt of Israel rode a bicycle around the world in 77 days. If Burt could have ridden along the equator, his average angular speed would have been $9.0 \times 10^{-7}$ rad/s. Now consider an object moving with this angular speed. How long would it take the object to reach an angular speed of $5.0 \times 10^{-6}$ rad/s if its angular acceleration was $7.5 \times 10^{-10}$ rad/s$^2$?

12. A coal-burning power plant in Kazakhstan has a chimney that is nearly 500 m tall. The radius of this chimney is 7.1 m at the base. Suppose a factory worker takes a 500.0 m run around the base of the chimney. If the worker starts with an angular speed of 0.40 rad/s and has an angular acceleration of $4.0 \times 10^{-3}$ rad/s$^2$, how long will the run take?
Advanced Topics

Problem E

TANGENTIAL SPEED

PROBLEM

In about 45 min, Nicholas Mason inflated a weather balloon using only lung power. If a fly, moving with a tangential speed of 5.11 m/s, were to make exactly 8 revolutions around this inflated balloon in 12.0 s, what would the balloon’s radius be?

SOLUTION

Given:

\[ v_t = 5.11 \text{ m/s} \]
\[ \Delta \theta = 8 \text{ rev} \]
\[ \Delta t = 12.0 \text{ s} \]

Unknown:

\[ \omega = ? \]
\[ r = ? \]

Use \( \Delta \theta \) and \( \Delta t \) to calculate the fly’s angular speed. Then rearrange the tangential speed equation to determine the balloon’s radius.

\[
\omega = \frac{\Delta \theta}{\Delta t} = \frac{(8 \text{ rev}) \left( \frac{2\pi \text{ rad}}{\text{rev}} \right)}{12.0 \text{ s}} = 4.19 \text{ rad/s}
\]

\[
v_t = r \omega
\]

\[
r = \frac{v_t}{\omega} = \frac{5.11 \text{ m/s}}{4.19 \text{ rad/s}} = 1.22 \text{ m}
\]

ADDITIONAL PRACTICE

1. The world’s tallest columns, which stand in front of the Education Building in Albany, New York, are each 30 m tall. If a fly circles a column with an angular speed of 4.44 rad/s, and its tangential speed is 4.44 m/s, what is the radius of the column?

2. The longest dingoproof wire fence stretches across southeastern Australia. Suppose this fence were to have a circular shape. A rancher driving around the perimeter of the fence with a tangential speed of 16.0 m/s has an angular speed of \( 1.82 \times 10^{-5} \) rad/s. What is the fence’s radius and length (circumference)?

3. The smallest self-sustaining gas turbine has a tiny wheel that can rotate at \( 5.24 \times 10^3 \) rad/s. If the wheel rim’s tangential speed is 131 m/s, what is the wheel’s radius?

4. Earth’s average tangential speed around the sun is about 29.7 km/s. If Earth’s average orbital radius is \( 1.50 \times 10^8 \) km, what is its angular orbital speed in rad/s?
5. Two English engineers designed a ridable motorcycle that was less than 12 cm long. The front wheel’s diameter was only 19.0 mm. Suppose this motorcycle was ridden so that the front wheel had an angular speed of 25.6 rad/s. What would the tangential speed of the front wheel’s rim have been?
Advanced Topics

Problem F

TANGENTIAL ACCELERATION

**PROBLEM**
In 1988, Stu Cohen made his kite perform 2911 figure eights in just 1 h. If the kite made a circular “loop” with a radius of 1.5 m and had a tangential acceleration of 0.6 m/s², what was the kite’s angular acceleration?

**SOLUTION**
Given: \( r = 1.5 \text{ m} \) \( a_t = 0.6 \text{ m/s}^2 \)

Unknown: \( \alpha = ? \)

Apply the tangential acceleration equation, solving for angular acceleration.

\[
a_t = r\alpha \]

\[
\alpha = \frac{a_t}{r} = \frac{0.6 \text{ m/s}^2}{1.5 \text{ m}} = 0.4 \text{ rad/s}^2
\]

**ADDITIONAL PRACTICE**

1. The world’s largest aquarium, at the EPCOT center in Orlando, Florida, has a radius of 32 m. Bicycling around this aquarium with a tangential acceleration of 0.20 m/s², what would your angular acceleration be?

2. Dale Lyons and David Pettifer ran the London marathon in less than 4 h while bound together at one ankle and one wrist. Suppose they made a turn with a radius of 8.0 m. If their tangential acceleration was \(-1.44 \text{ m/s}^2\), what was their angular acceleration?

3. In 1991, Timothy Badyna of the United States ran 10 km backward in just over 45 min. Suppose Badyna made a turn, so that his angular speed decreased \(2.4 \times 10^{-2} \text{ rad/s}\) in 6.0 s. If his tangential acceleration was \(-0.16 \text{ m/s}^2\), what was the radius of the turn?

4. Park Bong-tae of South Korea made 14 628 turns of a jump rope in 1.000 h. Suppose the rope’s average angular speed was gained from rest during the first turn and that the rope during this time had a tangential acceleration of 33.0 m/s². What was the radius of the rope’s circular path?

5. To “crack” a whip requires making its tip move at a supersonic speed. Kris King of Ohio achieved this with a whip 56.24 m long. If the tip of this whip moved in a circle and its angular speed increased from 6.00 rad/s to 6.30 rad/s in 0.60 s, what would the magnitude of the tip’s tangential acceleration be?

6. In 1986 in Japan, a giant top with a radius of 1.3 m was spun. The top remained spinning for over an hour. Suppose these people accelerated the top from rest so that the first turn was completed in 1.8 s. What was the tangential acceleration of points on the top’s rim?
Additional Practice A

**Given**

1. \(v_t = 0.17 \text{ m/s} \)
   \(a_c = 0.29 \text{ m/s}^2 \)

2. \(v_t = 465 \text{ m/s} \)
   \(a_c = 3.4 \times 10^{-2} \text{ m/s}^2 \)

3. \(r = \frac{58.4 \text{ cm}}{2} = 29.2 \text{ cm} \)
   \(a_c = 8.50 \times 10^{-2} \text{ m/s}^2 \)

4. \(r = \frac{12 \text{ cm}}{2} = 6.0 \text{ cm} \)
   \(a_c = 0.28 \text{ m/s}^2 \)

5. \(v_t = 7.85 \text{ m/s} \)
   \(r = 20.0 \text{ m} \)

6. \(\Delta t = 1.000 \text{ h} \)
   \(\Delta s = 47.112 \text{ km} \)
   \(r = 6.37 \times 10^3 \text{ km} \)

**Solution**

1. \[r = \frac{v_t^2}{a_c} = \frac{(0.17 \text{ m/s})^2}{0.29 \text{ m/s}^2} = 0.10 \text{ m} \]

2. \[r = \frac{v_t^2}{a_c} = \frac{(465 \text{ m/s})^2}{3.4 \times 10^{-2} \text{ m/s}^2} = 6.4 \times 10^{6} \text{ m} \]

3. \[v_t = \sqrt{r} = \sqrt{29.2 \times 10^{-2} \text{ m}(8.50 \times 10^{-2} \text{ m/s})^2} = 0.158 \text{ m/s} \]

4. \[v_t = \sqrt{r} = \sqrt{6.0 \times 10^{-2} \text{ m}(0.28 \text{ m/s})^2} = 0.13 \text{ m/s} \]

5. \[a_c = \frac{v_t^2}{r} = \frac{(7.85 \text{ m/s})^2}{20.0 \text{ m}} = 3.08 \text{ m/s}^2 \]

6. \[a_c = \frac{\Delta s}{\Delta t} = \frac{(47112 \text{ m})}{(1.000 \text{ h})} = \frac{28}{307 \text{ kg}} = 2.69 \times 10^{-3} \text{ m/s}^2 \]

Additional Practice B

1. \(m_1 = 235 \text{ kg} \)
   \(m_2 = 72 \text{ kg} \)
   \(r = 25.0 \text{ m} \)
   \(F_c = 1850 \text{ N} \)

   \[m_{\text{tot}} = m_1 + m_2 = 235 \text{ kg} + 72 \text{ kg} = 307 \text{ kg} \]

   \[F_c = m_{\text{tot}} a_c = m_{\text{tot}} \frac{v_t^2}{r} \]

   \[v_t = \sqrt{\frac{r}{m_{\text{tot}}} \left( \frac{25.0 \text{ m}}{1850 \text{ N}} \right)} = \frac{12.3 \text{ m/s}}{307 \text{ kg}} \]

2. \(m = 30.0 \text{ g} \)
   \(r = 2.4 \text{ m} \)
   \(F_T = 0.393 \text{ N} \)
   \(g = 9.81 \text{ m/s}^2 \)

   \[F_T = F_g + F_c = mg + m \frac{v_t^2}{r} \]

   \[v_t = \sqrt{\frac{r(F_T - mg)}{m}} = \sqrt{\frac{(2.4 \text{ m})(0.393 \text{ N} - 30.0 \times 10^{-3} \text{ kg})(9.81 \text{ m/s}^2)}{30.0 \times 10^{-3} \text{ kg}}} \]

   \[v_t = \frac{(2.4 \text{ m})(0.393 \text{ N} - 0.294 \text{ N})}{30.0 \times 10^{-3} \text{ kg}} = \frac{(2.4 \text{ m})(0.099 \text{ N})}{30.0 \times 10^{-3} \text{ kg}} \]

   \[v_t = 2.8 \text{ m/s} \]
### Given Values

1. \( v_t = 8.1 \text{ m/s} \)
   - \( r = 4.23 \text{ m} \)
   - \( m_1 = 25 \text{ g} \)
   - \( g = 9.81 \text{ m/s}^2 \)

2. \( v_t = 75.57 \text{ km/h} \)
   - \( m = 92.0 \text{ kg} \)
   - \( F_c = 12.8 \text{ N} \)

3. \( v_t = 12 \text{ m/s} \)
   - \( r = 446 \text{ m} \)
   - \( g = 9.81 \text{ m/s}^2 \)

4. \( v_t = 75.57 \text{ km/h} \)
   - \( m = 92.0 \text{ kg} \)
   - \( F_c = 12.8 \text{ N} \)

5. \( m = 75.0 \text{ kg} \)
   - \( r = 446 \text{ m} \)
   - \( v_t = 12 \text{ m/s} \)
   - \( g = 9.81 \text{ m/s}^2 \)

### Solution

\[
F_g = F_c \]
\[
m_1g = \frac{m_2v_t^2}{r} \]
\[
m_2 = \frac{m_1gr}{v_t^2} \]
\[
m_2 = \frac{(25 \times 10^{-3} \text{ kg})(9.81 \text{ m/s}^2)(4.23 \text{ m})}{(8.1 \text{ m/s})^2} = 1.6 \times 10^{-2} \text{ kg} \]

\[
F_c = \frac{m_1v_t^2}{r} \]
\[
r = \frac{m_1v_t^2}{F_c} = \frac{(92.0 \text{ kg})[(75.57 \text{ km/h})(10^3 \text{ m/km})(1 \text{ h}/3600 \text{ s})]^2}{12.8 \text{ N}} \]
\[
r = 3.17 \times 10^3 \text{ m} = 3.17 \text{ km} \]

\[
F_c = \frac{m_1v_t^2}{r} = \frac{(75.0 \text{ kg})(12 \text{ m/s})^2}{446 \text{ m}} = 24 \text{ N} \]
\[
F_T = F_c + mg = 24 \text{ N} + (75.0 \text{ kg})(9.81 \text{ m/s}^2) \]
\[
F_T = 24 \text{ N} + 780 \text{ N} = 7.60 \times 10^2 \text{ N} \]

### Additional Practice C

1. \( r = 6.3 \text{ km} \)
   - \( F_g = 2.5 \times 10^{-2} \text{ N} \)
   - \( m_1 = 3.0 \text{ kg} \)
   - \( G = 6.673 \times 10^{-11} \text{Nm}^2/\text{kg}^2 \)

\[
m_2 = \frac{F_g^2}{Gm_1} = \frac{(2.5 \times 10^{-2} \text{ N})(6.3 \times 10^3 \text{ m})^2}{6.673 \times 10^{-11} \text{Nm}^2/\text{kg}^2(3.0 \text{ kg})} \]
\[
m_2 = 5.0 \times 10^5 \text{ kg} \]

2. \( m_1 = 3.08 \times 10^4 \text{ kg} \)
   - \( r = 1.27 \times 10^7 \text{ m} \)
   - \( F_g = 2.88 \times 10^{-16} \text{ N} \)
   - \( G = 6.673 \times 10^{-11} \text{Nm}^2/\text{kg}^2 \)

\[
m_2 = \frac{F_g^2}{Gm_1} = \frac{(2.88 \times 10^{-16} \text{ N})(1.27 \times 10^7 \text{ m})^2}{6.673 \times 10^{-11} \text{Nm}^2/\text{kg}^2(3.08 \times 10^4 \text{ kg})} \]
\[
m_2 = 2.26 \times 10^4 \text{ kg} \]

3. \( m_1 = 5.81 \times 10^4 \text{ kg} \)
   - \( r = 25.0 \text{ m} \)
   - \( F_g = 5.00 \times 10^{-7} \text{ N} \)
   - \( G = 6.673 \times 10^{-11} \text{Nm}^2/\text{kg}^2 \)

\[
m_2 = \frac{F_g^2}{Gm_1} = \frac{(5.00 \times 10^{-7} \text{ N})(25.0 \text{ m})^2}{6.673 \times 10^{-11} \text{Nm}^2/\text{kg}^2(5.81 \times 10^4 \text{ kg})} \]
\[
m_2 = 80.6 \text{ kg} \]
**Section Two—Problem Workbook Solutions**

### 4. Given

\( m_1 = 621 \text{ g} \)
\( m_2 = 65.0 \text{ kg} \)
\( F_g = 1.0 \times 10^{-12} \text{ N} \)
\( G = 6.673 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \)

**Solution**

\[ r = \sqrt[3]{\frac{G m_1 m_2}{F_g}} = \sqrt[3]{\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.621 \text{ kg})(65.0 \text{ kg})}{1.0 \times 10^{-12} \text{ N}}} = 52 \text{ m} \]

### 5. Given

\( m_1 = m_2 = 1.0 \times 10^8 \text{ kg} \)
\( F_g = 1.0 \times 10^{-3} \text{ N} \)
\( G = 6.673 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \)

**Solution**

\[ r = \sqrt[3]{\frac{G m_1 m_2}{F_g}} = \sqrt[3]{\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.0 \times 10^8 \text{ kg})^2}{1.0 \times 10^{-3} \text{ N}}} = 2.6 \times 10^4 \text{ m} = 26 \text{ km} \]

### 6. Given

\( m_1 = 621 \text{ g} \)
\( m_2 = 50.0 \text{ kg} \)
\( F_g = 1.30 \times 10^{-2} \text{ N} \)
\( G = 6.673 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \)

**Solution**

\[ F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(318)(5.98 \times 10^{24} \text{ kg})}{(1.0 \times 10^6 \text{ m})^2} = 1.0 \times 10^{-2} \text{ N} \]

### 7. Given

\( m_1 = 318 m_E \)
\( m_2 = 50.0 \text{ kg} \)
\( V_f = 1323 V_E \)
\( m_E = 5.98 \times 10^{24} \text{ kg} \)
\( V_E = 1.86 \times 10^7 \text{ m} \)
\( m_E = 6.673 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \)

**Solution**

If \( V_f = 1323 V_E \), then \( r_f = \frac{V_f}{r_E} \)

\[ F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(318)(5.98 \times 10^{24} \text{ kg})}{(1.0 \times 10^6 \text{ m})^2} = 1.30 \times 10^{-4} \text{ N} \]

### Additional Practice D

#### 1. Given

\( T = 88.643 \text{ s} \)
\( m = 6.42 \times 10^{23} \text{ kg} \)
\( r_m = 3.40 \times 10^8 \text{ m} \)

**Solution**

\[ r = \sqrt[4]{\frac{G m T^2}{4 \pi^2}} = \sqrt[4]{\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(6.42 \times 10^{23} \text{ kg})(88.643 \text{ s})^2}{4 \pi^2}} \]

\[ r = 2.04 \times 10^7 \text{ m} \]

\[ r_s = r - r_m = 2.04 \times 10^7 \text{ m} - 3.40 \times 10^6 \text{ m} = 1.70 \times 10^7 \text{ m} \]

#### 2. Given

\( T = 5.51 \times 10^5 \text{ s} \)
\( m = 1.25 \times 10^{22} \text{ kg} \)

**Solution**

\[ r = \sqrt[4]{\frac{G m T^2}{4 \pi^2}} = \sqrt[4]{\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.25 \times 10^{22} \text{ kg})(5.51 \times 10^5 \text{ s})^2}{4 \pi^2}} \]

\[ r = 1.86 \times 10^7 \text{ m} \]
Advanced Topics

Appendix J

Additional Practice A

<table>
<thead>
<tr>
<th>Given</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>**1. ( r = 10.0 \text{ km} )  ( \Delta q = +15.0 \text{ rad} ) ( \Delta s = r \Delta q = (10.0 \text{ km})(15.0 \text{ rad}) = 1.50 \times 10^1 \text{ km} )</td>
<td>The particle moves in the positive, or counterclockwise, direction around the neutron star’s “north” pole.</td>
</tr>
<tr>
<td>**2. ( \Delta \theta = 3(2\pi \text{ rad}) ) ( r = 6560 \text{ km} ) ( \Delta s = r \Delta q = (6560 \text{ km})(3)(2\pi \text{ rad}) = 1.24 \times 10^5 \text{ km} )</td>
<td></td>
</tr>
<tr>
<td>**3. ( r = \frac{1.40 \times 10^3 \text{ km}}{2} ) ( \Delta \theta = 1.72 \text{ rad} ) ( r_E = 6.37 \times 10^3 \text{ km} ) ( \Delta \theta_E = \frac{\Delta s}{r_E} = \frac{(1.20 \times 10^5 \text{ km})(1 \text{ rev/2\pi rad})}{6.37 \times 10^3 \text{ km}} = 3.00 \text{ rev, or 3.00 orbits} )</td>
<td></td>
</tr>
<tr>
<td>**4. ( \Delta \theta = 225 \text{ rad} ) ( \Delta s = 1.50 \times 10^6 \text{ km} ) ( \Delta s = r \Delta \theta = \frac{1.50 \times 10^6 \text{ km}}{225 \text{ rad}} = 6.67 \times 10^3 \text{ km} )</td>
<td></td>
</tr>
<tr>
<td>**5. ( r = 5.8 \times 10^7 \text{ km} ) ( \Delta s = 1.5 \times 10^8 \text{ km} ) ( \Delta \theta = \frac{\Delta s}{r} = \frac{1.5 \times 10^8 \text{ km}}{5.8 \times 10^7 \text{ km}} = 2.6 \text{ rad} )</td>
<td></td>
</tr>
<tr>
<td>**6. ( \Delta s = -1.79 \times 10^4 \text{ km} ) ( \Delta \theta = \frac{\Delta s}{r} = \frac{-1.79 \times 10^4 \text{ km}}{6.37 \times 10^3 \text{ km}} = -2.81 \text{ rad} )</td>
<td></td>
</tr>
</tbody>
</table>

Additional Practice B

<table>
<thead>
<tr>
<th>Given</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>**1. ( r = 1.82 \text{ m} ) ( \omega_{\text{avg}} = 1.00 \times 10^{-1} \text{ rad/s} ) ( \Delta t = 60.0 \text{ s} ) ( \Delta \theta = \omega_{\text{avg}} \Delta t = (1.00 \times 10^{-1} \text{ rad/s})(60.0 \text{ s}) = 6.00 \text{ rad} )</td>
<td></td>
</tr>
<tr>
<td>**2. ( \Delta t = 120 \text{ s} ) ( \omega_{\text{avg}} = 0.40 \text{ rad/s} ) ( \Delta \theta = \omega_{\text{avg}} \Delta t = (0.40 \text{ rad/s})(120 \text{ s}) = 48 \text{ rad} )</td>
<td></td>
</tr>
<tr>
<td>**3. ( r = 30.0 \text{ m} ) ( \Delta s = 5.0 \times 10^3 \text{ m} ) ( \Delta \theta = \frac{\Delta s}{r} = \frac{5.0 \times 10^3 \text{ m}}{(30.0 \text{ m})(120 \text{ s})} = 0.14 \text{ rad/s} )</td>
<td></td>
</tr>
</tbody>
</table>
4. \( \Delta \theta = 16 \text{ rev} \)
\( \Delta t = 4.5 \text{ min} \)

\[ \omega_{av} = \frac{\Delta \theta}{\Delta t} = \frac{16 \text{ rev}}{(4.5 \text{ min})(60 \text{ s/min})} = 0.37 \text{ s} \]

5. \( \omega_{av} = \frac{2\pi \text{ rad}}{24 \text{ h}} \)
\( \Delta \theta = 0.262 \text{ rad} \)

\[ \Delta t = \frac{\Delta \theta}{\omega_{av}} = \frac{0.262 \text{ rad}}{\frac{2\pi \text{ rad}}{24 \text{ h}}} = 1.00 \text{ h} \]

6. \( r = 2.00 \text{ m} \)
\( \Delta s = 1.70 \times 10^2 \text{ km} \)
\( \omega_{av} = 5.90 \text{ rad/s} \)

\[ \Delta t = \frac{\Delta s}{r \omega_{av}} = \frac{1.70 \times 10^5 \text{ m}}{(2.00 \text{ m})(5.90 \text{ rad/s})} = 1.44 \times 10^4 \text{ s} = 4.00 \text{ h} \]

### Additional Practice C

1. \( \alpha_{av} = 2.0 \text{ rad/s}^2 \)
\( \omega_1 = 0 \text{ rad/s} \)
\( \omega_2 = 9.4 \text{ rad/s} \)

\[ \Delta t = \frac{\omega_2 - \omega_1}{\alpha_{av}} = \frac{9.4 \text{ rad/s} - 0 \text{ rad/s}}{2.0 \text{ rad/s}^2} = 4.7 \text{ s} \]

2. \( \Delta t = 9.83 \text{ h} \)
\( \alpha_{av} = -3.0 \times 10^{-8} \text{ rad/s}^2 \)
\( \omega_2 = 0 \text{ rad/s} \)

\[ \Delta t = \frac{\omega_2 - \omega_1}{\alpha_{av}} = \frac{0.00 \text{ rad/s} - 1.78 \times 10^{-4} \text{ rad/s}}{-3.0 \times 10^{-8} \text{ rad/s}^2} = 5.9 \times 10^3 \text{ s} \]

3. \( \omega_1 = 2.00 \text{ rad/s} \)
\( \omega_2 = 3.15 \text{ rad/s} \)
\( \Delta t = 3.6 \text{ s} \)

\[ \alpha_{av} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{3.15 \text{ rad/s} - 2.00 \text{ rad/s}}{3.6 \text{ s}} = \frac{1.15 \text{ rad/s}}{3.6 \text{ s}} = 0.32 \text{ rad/s}^2 \]

4. \( \omega_1 = 8.0 \text{ rad/s} \)
\( \omega_2 = 3\omega_1 = 24 \text{ rad/s} \)
\( \Delta t = 25 \text{ s} \)

\[ \alpha_{av} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{24 \text{ rad/s} - 8.0 \text{ rad/s}}{25 \text{ s}} = \frac{16 \text{ rad/s}}{25 \text{ s}} = 0.64 \text{ rad/s}^2 \]

5. \( \Delta t = 365 \text{ days} \)
\( \Delta \theta_1 = 2\pi \text{ rad} \)
\( \alpha_{av} = 6.05 \times 10^{-13} \text{ rad/s}^2 \)
\( \Delta t_2 = 12.0 \text{ days} \)

\[ \omega_2 = \omega_1 + \alpha_{av}\Delta t_2 = 1.99 \times 10^{-7} \text{ rad/s} + (6.05 \times 10^{-13} \text{ rad/s}^2)(12.0 \text{ days}) \]
\[ = 8.26 \times 10^{-7} \text{ rad/s} \]

6. \( \omega_1 = 0 \text{ rad/s} \)
\( \alpha_{av} = 0.800 \text{ rad/s}^2 \)
\( \Delta t = 8.40 \text{ s} \)

\[ \omega_2 = \omega_1 + \alpha_{av}\Delta t = 0 \text{ rad/s} + (0.800 \text{ rad/s}^2)(8.40 \text{ s}) = 6.72 \text{ rad/s} \]
Additional Practice D

**Given**

1. \( \omega_i = 5.0 \text{ rad/s} \)
   \( \alpha = 0.60 \text{ rad/s}^2 \)
   \( \Delta t = 0.50 \text{ min} \)

\( \omega_f = \omega_i + \alpha \Delta t \)

\( \omega_f = 5.0 \text{ rad/s} + (0.60 \text{ rad/s}^2)(0.50 \text{ min})(60.0 \text{ s/min}) \)

\( \omega_f = 5.0 \text{ rad/s} + 18 \text{ rad/s} \)

\( \omega_f = 23 \text{ rad/s} \)

2. \( \alpha = 1.0 \times 10^{-10} \text{ rad/s}^2 \)
   \( \Delta t = 12 \text{ h} \)

\( \omega_i = \frac{2\pi \text{ rad}}{27.3 \text{ days}} \)

\( \omega_f = \omega_i + \alpha \Delta t = 2.66 \times 10^{-6} \text{ rad/s} + (1.0 \times 10^{-10} \text{ rad/s}^2)(12 \text{ h})(3600 \text{ s/h}) \)

\( \omega_f = 2.66 \times 10^{-6} \text{ rad/s} + 4.3 \times 10^{-6} \text{ rad/s} = 7.0 \times 10^{-6} \text{ rad/s} \)

3. \( r = \frac{43 \text{ m}}{2\pi \text{ rad}} \)
   \( \omega_i = 0 \text{ rad/s} \)
   \( \Delta s = 160 \text{ m} \)
   \( \alpha = 5.0 \times 10^{-2} \text{ rad/s}^2 \)

\( \Delta \theta = \frac{\Delta s}{r} \)

\( \omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta = \omega_i^2 + \frac{2\alpha \Delta s}{r} \)

\( \omega_f = \sqrt{\omega_i^2 + 2\alpha \Delta s} = \sqrt{(0 \text{ rad/s})^2 + \frac{(2)(5.00 \times 10^{-2} \text{ rad/s}^2)(160 \text{ m})}{43 \text{ m}}} \)

\( \omega_f = 1.5 \text{ rad/s} \)

4. \( \Delta s = 52.5 \text{ m} \)
   \( \alpha = -3.2 \times 10^{-5} \text{ rad/s}^2 \)

\( \omega_f = 0.080 \text{ rad/s} \)

\( r = 8.0 \text{ cm} \)

\( \Delta \theta = \frac{\Delta s}{r} \)

\( \omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta = \omega_i^2 + \frac{2\alpha \Delta s}{r} \)

\( \omega_f = \sqrt{\omega_i^2 - \frac{2\alpha \Delta s}{r}} = \sqrt{(0.080 \text{ rad/s})^2 - \frac{(2)(-3.2 \times 10^{-5} \text{ rad/s}^2)(52.5 \text{ m})}{8.0 \times 10^{-2} \text{ m}}} \)

\( \omega_f = \sqrt{6.4 \times 10^{-3} \text{ rad}^2/\text{s}^2 + 4.2 \times 10^{-2} \text{ rad}^2/\text{s}^2} = \sqrt{4.8 \times 10^{-2} \text{ rad}^2/\text{s}^2} \)

\( \omega_f = 0.22 \text{ rad/s} \)

5. \( r = 3.0 \text{ m} \)

\( \omega_i = 0.820 \text{ rad/s} \)

\( \omega_f = 0.360 \text{ rad/s} \)

\( \Delta s = 20.0 \text{ m} \)

\( \Delta \theta = \frac{\Delta s}{r} \)

\( \alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta \theta} = \frac{\omega_f^2 - \omega_i^2}{2\frac{\Delta s}{r}} = \frac{(0.360 \text{ rad/s})^2 - (0.820 \text{ rad/s})^2}{2\frac{20.0 \text{ m}}{3.0 \text{ m}}} \)

\( \alpha = \frac{0.130 \text{ rad}^2/\text{s}^2 - 0.672 \text{ rad}^2/\text{s}^2}{2\frac{20.0 \text{ m}}{3.0 \text{ m}}} = -0.542 \text{ rad}^2/\text{s}^2 \)

\( \alpha = \frac{-0.524 \text{ rad}^2/\text{s}^2}{2\frac{20.0 \text{ m}}{3.0 \text{ m}}} \)

\( \alpha = -4.1 \times 10^{-5} \text{ rad/s}^2 \)
Appendix J–4

**Given**

6. \( r = 1.0 \text{ km} \)
   \( \omega_i = 5.0 \times 10^{-3} \text{ rad/s} \)
   \( \Delta t = 14.0 \text{ min} \)
   \( \Delta \theta = 2\pi \text{ rad} \)

\[ \Delta \theta = \omega_i \Delta t + \frac{1}{2} \alpha \Delta t^2 \]

\[ \alpha = \frac{2(\Delta \theta - \omega_i \Delta t)}{\Delta t^2} = \frac{(2)[2\pi \text{ rad} - (5.0 \times 10^{-3} \text{ rad/s})(14.0 \text{ min})(60 \text{ s/min})]}{(14.0 \text{ min})(60 \text{ s/min})^2} \]

\[ \alpha = \frac{(2)(6.3 \text{ rad} - 4.2 \text{ rad})}{[(14.0 \text{ min})(60 \text{ s/min})]^2} = \frac{(2)(2.1 \text{ rad})}{[(14.0 \text{ min})(60 \text{ s/min})]^2} = 6.0 \times 10^{-6} \text{ rad/s}^2 \]

7. \( \omega_i = 7.20 \times 10^{-2} \text{ rad/s} \)
   \( \Delta \theta = 12.6 \text{ rad} \)
   \( \Delta t = 4 \text{ min}, 22 \text{ s} = 262 \text{ s} \)

\[ \Delta \theta = \omega_i \Delta t + \frac{1}{2} \alpha \Delta t^2 \]

\[ \alpha = \frac{2(\Delta \theta - \omega_i \Delta t)}{\Delta t^2} = \frac{(2)[12.6 \text{ rad} - (7.20 \times 10^{-2} \text{ rad/s})(262 \text{ s})]}{(262 \text{ s})^2} \]

\[ \alpha = \frac{(2)(12.6 \text{ rad} - 18.9 \text{ rad})}{(262 \text{ s})^2} = \frac{(2)(-6.3 \text{ rad/s})}{(262 \text{ s})^2} = -1.8 \times 10^{-4} \text{ rad/s}^2 \]

8. \( \omega_i = 27.0 \text{ rad/s} \)
   \( \omega_f = 32.0 \text{ rad/s} \)
   \( \Delta t = 6.83 \text{ s} \)

\[ \alpha_{\text{avg}} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{32.0 \text{ rad/s} - 27.0 \text{ rad/s}}{6.83 \text{ s}} = \frac{5.0 \text{ rad/s}}{6.83 \text{ s}} \]

\[ \alpha_{\text{avg}} = 0.73 \text{ rad/s}^2 \]

9. \( \alpha = 2.68 \times 10^{-5} \text{ rad/s}^2 \)
   \( \Delta t = 120.0 \text{ s} \)
   \( \omega_i = \frac{2\pi \text{ rad}}{12} \)

\[ \Delta \theta = \omega_i \Delta t + \frac{1}{2} \alpha \Delta t^2 \]

\[ \Delta \theta = \left( \frac{2\pi \text{ rad}}{12 \text{ h}} \right)(1 \text{ h/3600 s})(120.0 \text{ s}) + \frac{1}{2}(2.68 \times 10^{-5} \text{ rad/s}^2)(120.0 \text{ s})^2 \]

\[ \Delta \theta = 1.7 \times 10^{-2} \text{ rad} + 1.93 \times 10^{-1} \text{ rad} = 0.210 \text{ rad} \]

10. \( \omega_f = 6.0 \times 10^{-3} \text{ rad/s} \)
    \( \omega_f = 3 \omega_f = 18 \times 10^{-3} \text{ rad/s} \)
    \( \alpha = 2.5 \times 10^{-4} \text{ rad/s}^2 \)

\[ \Delta \theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} \]

\[ \Delta \theta = \frac{(18 \times 10^{-3} \text{ rad/s})^2 - (6.0 \times 10^{-3} \text{ rad/s})^2}{(2)(2.5 \times 10^{-4} \text{ rad/s}^2)} = \frac{3.2 \times 10^{-4} \text{ rad}^2/s^2 - 3.6 \times 10^{-5} \text{ rad}^2/s^2}{(2)(2.5 \times 10^{-4} \text{ rad/s}^2)} \]

\[ \Delta \theta = \frac{2.8 \times 10^{-4} \text{ rad}^2/s^2}{5.0 \times 10^{-4} \text{ rad}^2/s^2} = 0.56 \text{ rad} \]

11. \( \omega_i = 9.0 \times 10^{-7} \text{ rad/s} \)
    \( \omega_f = 5.0 \times 10^{-6} \text{ rad/s} \)
    \( \alpha = 7.5 \times 10^{-10} \text{ rad/s}^2 \)

\[ \Delta t = \frac{\omega_f - \omega_i}{\alpha} \]

\[ \Delta t = \frac{5.0 \times 10^{-6} \text{ rad/s} - 9.0 \times 10^{-7} \text{ rad/s}}{7.5 \times 10^{-10} \text{ rad/s}^2} = \frac{4.1 \times 10^{-6} \text{ rad/s}}{7.5 \times 10^{-10} \text{ rad/s}^2} \]

\[ \Delta t = 5.5 \times 10^3 \text{ s} = 1.5 \text{ h} \]
**Additional Practice E**

1. \( \omega = 4.44 \text{ rad/s} \)
   \( \nu_t = 4.44 \text{ m/s} \)
   \[ r = \frac{\nu_t}{\omega} = \frac{4.44 \text{ m/s}}{4.44 \text{ rad/s}} = 1.00 \text{ m} \]

2. \( \nu_t = 16.0 \text{ m/s} \)
   \( \omega = 1.82 \times 10^{-5} \text{ rad/s} \)
   \[ r = \frac{\nu_t}{\omega} = \frac{16.0 \text{ m/s}}{1.82 \times 10^{-5} \text{ rad/s}} = 8.79 \times 10^5 \text{ m} = 879 \text{ km} \]
   Circumference = \( 2\pi r = 2\pi (879 \text{ km}) = 5.52 \times 10^3 \text{ km} \)

3. \( \omega = 5.24 \times 10^5 \text{ rad/s} \)
   \( \nu_t = 131 \text{ m/s} \)
   \[ r = \frac{\nu_t}{\omega} = \frac{131 \text{ m/s}}{5.24 \times 10^5 \text{ rad/s}} = 2.50 \times 10^{-2} \text{ m} = 2.50 \text{ cm} \]

4. \( \nu_t = 29.7 \text{ km/s} \)
   \( r = 1.50 \times 10^8 \text{ km} \)
   \[ \omega = \frac{\nu_t}{r} = \frac{29.7 \text{ km/s}}{1.50 \times 10^8 \text{ km}} = 1.98 \times 10^{-7} \text{ rad/s} \]

5. \( r = \frac{19.0 \text{ mm}}{2} = 9.50 \text{ mm} \)
   \( \omega = 25.6 \text{ rad/s} \)
   \[ \nu_t = r \omega = (9.50 \times 10^{-3} \text{ m})(25.6 \text{ rad/s}) = 0.243 \text{ m/s} \]
**Additional Practice F**

### Given

1. \( r = 32 \text{ m} \)
   \( a_t = 0.20 \text{ m/s}^2 \)
   \[ a = \frac{a_t}{r} = \frac{0.20 \text{ m/s}^2}{32 \text{ m}} = 6.2 \times 10^{-3} \text{ rad/s}^2 \]

2. \( r = 8.0 \text{ m} \)
   \( a_t = -1.44 \text{ m/s}^2 \)
   \[ a = \frac{a_t}{r} = \frac{-1.44 \text{ m/s}^2}{8.0 \text{ m}} = -0.18 \text{ rad/s}^2 \]

3. \( \Delta \omega = -2.4 \times 10^{-2} \text{ rad/s} \)
   \( \Delta t = 6.0 \text{ s} \)
   \[ a = \frac{\Delta \omega}{\Delta t} = \frac{-2.4 \times 10^{-2} \text{ rad/s}}{6.0 \text{ s}} = -4.0 \times 10^{-3} \text{ rad/s}^2 \]
   \[ r = a = \frac{-0.16 \text{ m/s}^2}{-4.0 \times 10^{-1} \text{ rad/s}} = 4.0 \times 10^{-1} \text{ m} \]

4. \( \Delta \theta' = 14,628 \text{ turns} \)
   \( \Delta t' = 1.000 \text{ h} \)
   \( a_t = 33.0 \text{ m/s}^2 \)
   \( \omega_i = 0 \text{ rad/s} \)
   \( \omega_f = 6.30 \text{ rad/s} \)
   \[ \Delta \theta = 2 \pi \text{ rad} \]

   \[ r = \frac{a_t}{\alpha} = \frac{33.0 \text{ m/s}^2}{2(14,628 \text{ turns})(2 \pi \text{ rad/turn})} - 0 \text{ rad/s}^2 = 0.636 \text{ m} \]

5. \( r = 56.24 \text{ m} \)
   \( \omega_i = 6.00 \text{ rad/s} \)
   \( \omega_f = 6.30 \text{ rad/s} \)
   \[ \Delta \theta = 2 \pi \text{ rad} \]
   \[ \Delta t = 0.60 \text{ s} \]

   \[ a_t = \frac{r \omega_f - \omega_i}{\Delta t} = \frac{56.24 \text{ m}(6.30 \text{ rad/s} - 6.00 \text{ rad/s})}{0.60 \text{ s}} = (56.24 \text{ m})(0.30 \text{ rad/s}) \]

   \[ a_t = 58 \text{ m/s}^2 \]

### Solutions